Calculating The Ex-Post Cost of Capital
The Actual and Adjusted Share Closing Price

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In this white paper we will calculate a share's actual closing price (includes the effect of dividends and dilution) and adjusted closing price (excludes the effect of dividends and dilution). To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are given the following model assumptions...

Table 1: Model Assumptions

Assumption	Value
Annualized cash flow at time zero	100,000
Annual risk-adjusted discount rate	12.00%
Annual cash flow growth rate	5.00%
Share count at time zero	10,000
Share grant in month 3 (zero proceeds)	5,000
Share grant in month 9 (zero proceeds)	2,500

Question 1: What is monthly cash flow, cash flow growth rate, discount rate and dividend yield?
Question 2: What is share actual closing price at the end of the year?
Question 3: Reconcile the change in share price over the annual time period.
Question 4: Calculate the share's adjusted closing price.

Share Price Equations - Actual Closing Price

We will define the discrete-time period length to be one month. The equation to convert an annual rate to a monthly rate is...

$$\text{Periodic rate} = \left(1 + \text{Annual rate}\right)^{1/12} - 1 \tag{1}$$

We will define the variable g to be the discrete-time periodic cash flow growth rate and the variable k to be the discrete-time periodic discount rate. Using Equation (1) above, the equations for these two variables are...

$$g = \left(1 + \text{Annual cash flow growth rate}\right)^{1/12} - 1 \text{ ...and... } k = \left(1 + \text{Annual discount rate}\right)^{1/12} - 1 \tag{2}$$

We will define the variable C_0 to be periodic company free cash flow at time zero.

$$C_0 = \text{Annualized company cash flow at time zero} \times \frac{1}{12} \tag{3}$$

We will define the variable n to be the number of discrete-time periods and the variable C_n to be periodic company free cash flow at the end of time period n. Using Equations (2) and (3) above, the equation for expected company periodic cash flow is...

$$C_n = C_0\left(1 + g\right)^n \tag{4}$$

We will define the variable S_n to be share price at the end of period n and the variable Γ_n to be the number of shares outstanding at the end of period n. Using Equations (2) and (4) above, the equation for expected share price is...

$$S_n = \sum_{m=1}^{\infty} C_0\,\Gamma_n^{-1}\Big(1+g\Big)^n\Big(1+g\Big)^m\Big(1+k\Big)^{-m} \tag{5}$$

Note that we can rewrite Equation (5) above as...

$$S_n = C_0\,\Gamma_n^{-1}\Big(1+g\Big)^n \sum_{m=1}^{\infty}\theta^m \;\text{...where...}\; \theta = \frac{1+g}{1+k} \tag{6}$$

Total return consists of dividends plus capital gains. If total return is k (risk-adjusted discount rate) and the cash flow growth rate is g, then the dividend yield must be $k-g$. If we define the variable d to be the dividend yield then the following equation holds...

$$\text{if... } d = k - g \text{ ...then... } g = k - d \tag{7}$$

Using the definition in Equation (7) above, we can rewrite Equation (6) above as...

$$S_n = C_0\,\Gamma^{-1}\Big(1+k-d\Big)^n \sum_{m=1}^{\infty}\theta^m \;\text{...where...}\; \theta = \frac{1+k-d}{1+k} \tag{8}$$

The equation for a geometric series is... [1]

$$\sum_{m=1}^{\infty}\theta^m = \frac{\theta}{1-\theta} \;\text{...given that...}\; 0 < \theta < 1 \tag{9}$$

Using Equation (9) above, we can rewrite Equation (8) above as...

$$S_n = C_0\,\Gamma_n^{-1}\Big(1+k-d\Big)^n \frac{\theta}{1-\theta} \tag{10}$$

Using Appendix Equation (22) below, we can rewrite Equation (10) above as...

$$S_n = C_0\,\Gamma_n^{-1}\Big(1+k-d\Big)^{n+1}\Big/ d \tag{11}$$

Using Equation (11) above, the equation for share price at time n as a function of share price at time $n-1$ is...

$$\text{if... } S_{n-1} = C_0\,\Gamma_{n-1}^{-1}\Big(1+k-d\Big)^n\Big/ d \text{ ...then... } S_n = S_{n-1}\Big(1+k-d\Big)\frac{\Gamma_{n-1}}{\Gamma_n} \tag{12}$$

Share Price Equations - Adjusted Closing Price

The textual equation for the periodic change in share price is...

$$\text{End share price} = \text{Begin share price} + \text{Total return} - \text{Dividend paid} - \text{Dilution} \tag{13}$$

Using Appendix Equation (24) below, the equation for the periodic change in share price is...

$$S_n - S_{n-1} = \left(\frac{\Gamma_{n-1}-\Gamma_n}{\Gamma_n}\right)\Big(1+k-d\Big)S_{n-1} + k\,S_{n-1} - d\,S_{n-1} \tag{14}$$

Using Equations (13) and (14) above, the components of share return are...

$$\text{Total return} = k\,S_{n-1} \;\Big|\; \text{Dividend paid} = d\,S_{n-1} \;\Big|\; \text{Dilution} = \frac{\Gamma_n - \Gamma_{n-1}}{\Gamma_n}\Big(1+k-d\Big)S_{n-1} \tag{15}$$

We will define the variable V_n to be the adjusted closing share price at the end of period n. As noted in the introduction, the share's adjusted closing price assumes dividends are reinvested in the shares (dividend yield is zero) and the effects of dilution are eliminated. Using Equations (14) and (15) above, the equation for the adjusted change in share price is...

$$V_n - V_{n-1} = k\,V_{n-1} \tag{16}$$

Using Equation (16) above, the equation for adjusted closing price is...

$$\text{if... } V_n = (1+k)\,V_{n-1} \text{ ...then... } V_{n-1} = (1+k)^{-1}\,V_n \tag{17}$$

Note that to calculate adjusted share prices prior to time period n we have to work backwards from the current actual closing price.

Answers to our Hypothetical Problem

Question 1: What is monthly cash flow, cash flow growth rate, discount rate and dividend yield?

Using Equations (2), (3), (7) above and the data in Table 1 above, the answers to the question are...

$$k = \Bigl(1 + 0.12\Bigr)^{1/12} - 1 = 0.0095 \;\Big|\; g = \Bigl(1 + 0.12\Bigr)^{1/12} - 1 = 0.0041 \;\Big|\; d = 0.0095 - 0.0041 = 0.0054 \tag{18}$$

$$C_0 = 100,000 \times \frac{1}{12} = 8,333 \tag{19}$$

Question 2: What is share actual closing price at the end of the year?

Using Equation (11) above, the answer to the question is...

$$S_{12} = 8,333 \times 17,500^{-1} \times \Bigl(1 + 0.0095 - 0.0054\Bigr)^{13} \Big/ 0.0054 = 92.72 \tag{20}$$

Question 3: Reconcile the change in share price over the annual time period.

The answer to the question is...

Month Number	Share Count	Total CF	Per Sh CF	Actual Closing Price Reconciliation Begin	TReturn	Dividends	Dilution	End
0	8,333	10,000	0.8333	154.53	–	–	–	154.53
1	8,367	10,000	0.8367	154.53	1.47	-0.84	0.00	155.16
2	8,401	10,000	0.8401	155.16	1.47	-0.84	0.00	155.79
3	8,436	15,000	0.5624	155.79	1.48	-0.84	-52.14	104.28
4	8,470	15,000	0.5647	104.28	0.99	-0.56	0.00	104.71
5	8,504	15,000	0.5670	104.71	0.99	-0.57	0.00	105.14
6	8,539	15,000	0.5693	105.14	1.00	-0.57	0.00	105.56
7	8,574	15,000	0.5716	105.56	1.00	-0.57	0.00	105.99
8	8,609	15,000	0.5739	105.99	1.01	-0.57	0.00	106.43
9	8,644	17,500	0.4939	106.43	1.01	-0.58	-15.27	91.59
10	8,679	17,500	0.4960	91.59	0.87	-0.50	0.00	91.97
11	8,714	17,500	0.4980	91.97	0.87	-0.50	0.00	92.34
12	8,750	17,500	0.5000	92.34	0.88	-0.50	0.00	92.72

Using Row 3 as an example:

$$\begin{aligned}
\text{Begin price} &= 8,333 \times 10,000^{-1} \times (1 + 0.0095 - 0.0054)^{3} \Big/ 0.0054 = 155.79 \\
\text{Total return} &= 0.0095 \times 155.79 = 1.48 \\
\text{Dividend} &= 0.0054 \times 155.79 = 0.84 \\
\text{Dilution} &= \frac{15,000 - 10,000}{15,000} \times (1 + 0.0095 - 0.0054)^{4} \times 155.79 = 52.14
\end{aligned} \tag{21}$$

The calculations above use Equations (11) and (15) above...

Question 4: Calculate the share's adjusted closing price.

Using Equation (17) above and the closing share price at time n (Equation (20) above), the answer to the question is...

Month Number	Adjusted Close
0	82.78
1	83.57
2	84.36
3	85.16
4	85.97
5	86.79
6	87.61
7	88.44
8	89.28
9	90.13
10	90.98
11	91.85
12	92.72

Appendix

A. The solution to the following equation is...

$$\begin{aligned} S_n &= C_0 \left(1+k-d\right)^n \frac{1+k-d}{1+k} \bigg/ 1 - \frac{1+k-d}{1+k} \\ &= C_0 \left(1+k-d\right)^n \frac{1+k-d}{1+k} \bigg/ \frac{1+k-1-k+d}{1+k} \\ &= C_0 \left(1+k-d\right)^n (1+k-d) \Big/ d \\ &= C_0 \left(1+k-d\right)^{n+1} \Big/ d \end{aligned} \tag{22}$$

B. Using Equation (11) above, the solution to the following equation is...

$$\begin{aligned} S_{n+m} &= C_0\, \Gamma_n^{-1} \left(1+k-d\right)^{n+1+m} \Big/ d \\ &= C_0\, \Gamma^{-1} \left(1+k-d\right)^{n+1} \left(1+k-d\right)^m \Big/ d \\ &= S_n \left(1+k-d\right)^m \end{aligned} \tag{23}$$

C. Using Equation (12) above, the solution to the following equation is...

$$\begin{aligned} S_n &= S_{n-1} \left(1+k-d\right) \frac{\Gamma_{n-1}}{\Gamma_n} \\ S_n &= S_{n-1} \frac{\Gamma_{n-1}}{\Gamma_n} + k\, S_{n-1} \frac{\Gamma_{n-1}}{\Gamma_n} - d\, S_{n-1} \frac{\Gamma_{n-1}}{\Gamma_n} \\ S_n &= S_{n-1} \left(\frac{\Gamma_{n-1}}{\Gamma_n} - 1\right) \left(1+k-d\right) + S_{n-1} + k\, S_{n-1} - d\, S_{n-1} \\ S_n - S_{n-1} &= \left(\frac{\Gamma_{n-1} - \Gamma_n}{\Gamma_n}\right) \left(1+k-d\right) S_{n-1} + k\, S_{n-1} - d\, S_{n-1} \end{aligned} \tag{24}$$

References

[1] Gary Schurman, *Polylogarithms Of Order Zero,* May, 2019.